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We conclude that the adopted value of the precession is not widely wrong; at the same time, if our theory is correct, we must not trust too far the present methods of calculating it. They are only approximately (and one might almost say accidentally) correct.

9. *Summary and Results.*

The Greenwich-Groombridge and Greenwich-Carrington proper motions strongly support Kapteyn's hypothesis of two star-drifts.

The additional results arrived at in this paper (which, of course, depend on the truth of this hypothesis) are:—

1. The numbers of the stars belonging to the two drifts are nearly equal.
2. One drift is moving relatively to the sun, with a speed between three and four times that of the other.
3. The proportion of stars belonging to each drift is about the same in every part of the sky and at every distance here dealt with.
4. The magnitudes of the stars belonging to the two drifts are about the same.
5. The spectral types of the stars seem to differ to some extent in the two drifts. But the evidence is not conclusive.
6. The results apply to all stars down to magnitude 9·5.

Royal Observatory, Greenwich:
1906 *August.*

On the Effects of Radiation on the Motion of Comets (*Second Note*). By H. C. Plummer, M.A.

1. Professor Poynting has recently pointed out to me the existence of an additional term in the forces which are due to the action of solar radiation on a small black body. The term has no effect when the motion of the particle is circular, but it becomes necessary to amend the theory which was given in §§ 5–8 of my former paper (*Monthly Notices*, vol. lxv. p. 232). This is due to the fact that when the body is moving with a radial velocity $\frac{dr}{dt}$ the light pressure is altered in the ratio $1 : 1 - \frac{1}{U} \cdot \frac{dr}{dt}$ where U is the velocity of light. The effect of the radial component of the motion was before omitted.

2. The radial accelerations of the particle are

(i.) that due to gravitation,

$$R_g = -\frac{\mu}{r^2}$$

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(ii.) that caused by a "static" pressure due to radiation,

$$R_s = +\frac{\mu'}{r^2}$$

(iii.) that caused by a "kinetic" pressure due to radiation (this is the term now introduced),

$$R_k = -\frac{1}{U}\frac{dr}{dt}\cdot\frac{\mu'}{r^2}$$

There is also a tangential retardation Tv/r^2, which may be resolved into

(iv.) a radial component

$$R_t = -\frac{T}{r^2}\cdot\frac{dr}{dt}$$

(v.) a transversal component

$$V_t = -\frac{T}{r}\cdot\frac{d\theta}{dt}$$

Now if S is the solar constant at the Earth's distance b from the Sun, and a is the radius of the spherical particle of density ρ,

$$T = Sb^2/4U^2a\rho \quad . \quad . \quad . \quad . \quad (1)$$

$$\mu' = 3Sb^2/4Ua\rho = 3UT \quad . \quad . \quad . \quad (2)$$

Hence the complete radial component is

$$\Sigma R = -\frac{\mu - \mu'}{r^2} - \frac{4T}{r^2}\cdot\frac{dr}{dt} \quad . \quad . \quad . \quad (3)$$

3. One equation of motion remains as before

$$\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = rV_t = -T\frac{d\theta}{dt}$$

or on integration

$$r^2\frac{d\theta}{dt} \equiv h = \frac{T}{c}(1 - c\theta) \quad . \quad . \quad . \quad (4)$$

where c is a small constant of the same order as T. The other equation of motion is

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \Sigma R$$

which becomes on eliminating t and putting $u = 1/r$

$$h^2u^2\left(\frac{d^2u}{d\theta^2} + u\right) + hu^2\frac{dh}{d\theta}\cdot\frac{du}{d\theta} + \Sigma R = 0$$

By (3) and (4) this is reduced to

$$\frac{d^2u}{d\theta^2} + u + \frac{3c}{1 - c\theta}\frac{du}{d\theta} = \frac{(\mu - \mu')c^2}{(1 - c\theta)^2T^2} \quad . \quad . \quad (5)$$

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which differs from the corresponding equation of the former paper by the addition of the third term on the left.

4. Neglecting second-order terms we replace (5) by

$$\frac{d^2u}{d\theta^2}+3c\frac{du}{d\theta}+u=(\mu-\mu')c^2\mathrm{T}^{-2}(1+2c\theta)$$

the integral of which, to the same order of approximation, is

$$u=(\mu-\mu')c^2\mathrm{T}^{-2}\{1+2c\theta+e\cos(\theta-\gamma)-\tfrac{3}{2}ce\theta\cos(\theta-\gamma)\} \qquad (6)$$

Let the osculating orbit at the point θ be

$$u=l^{-1}\{1+e'\cos(\theta-\gamma')\}$$

Since r, $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$ are the same for the two curves, u, $\frac{du}{d\theta}$ and h are also the same. Therefore by (4)

$$l=\frac{h^2}{\mu-\mu'}=\frac{\mathrm{T}^2}{c^2}\,\frac{(1-c\theta)^2}{\mu-\mu'} \quad . \quad . \quad . \quad (7)$$

where θ is to be considered constant at the point. Hence we have

$$\begin{aligned}\frac{u\mathrm{T}^2}{(\mu-\mu')c^2}&=1+2c\theta+e\cos(\theta-\gamma)-\tfrac{3}{2}ce\theta\cos(\theta-\gamma)\\&=(1+2c\theta)\{1+e'\cos(\theta-\gamma')\}\end{aligned}$$

and also $\frac{du}{d\theta}$ the same for both expressions, provided that in the second we regard $c\theta$ as constant. Hence, still neglecting terms beyond the first order,

$$\begin{aligned}e'\cos(\theta-\gamma')-e\cos(\theta-\gamma)&=-\tfrac{7}{2}ce\theta\cos(\theta-\gamma)\\e'\sin(\theta-\gamma')-e\sin(\theta-\gamma)&=-\tfrac{7}{2}ce\theta\sin(\theta-\gamma)+\tfrac{3}{2}ce\cos(\theta-\gamma)-2c\end{aligned}$$

From these can be derived

$$\begin{aligned}\delta e&=-\tfrac{7}{2}ce\theta+\tfrac{3}{4}ce\sin 2(\theta-\gamma)-2c\sin(\theta-\gamma) \quad . \quad (8)\\e\,.\,\delta\gamma&=-\tfrac{3}{2}ce\cos^2(\theta-\gamma)+2c\cos(\theta-\gamma) \quad . \quad . \quad (9)\end{aligned}$$

where the increments refer to the osculating ellipse as compared with the ellipse obtained by suppressing the accents on e and γ.

5. The secular changes in the elements can now be deduced from (7), (8), and (9) by considering the effect of a complete revolution. The result of thus removing the periodic variations is clearly

$$\begin{aligned}\Delta l/l&=-4\pi c \quad . \quad . \quad . \quad . \quad (10)\\\Delta e/e&=-7\pi c \quad . \quad . \quad . \quad . \quad (11)\\\Delta\gamma&=0 \quad . \quad . \quad . \quad . \quad . \quad (12)\end{aligned}$$

Thus the result of the additional term in the action of solar radiation is by (11) to increase the secular change in the eccentricity in the ratio of 7 : 4.

Further,

$$\frac{\Delta a}{a}=\frac{\Delta l}{l}+\frac{2e\Delta e}{1-e^2}=-\frac{4+10e^2}{1-e^2}\cdot\pi c \quad . \quad . \quad (13)$$

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and

$$\frac{\Delta n}{n} = -\tfrac{3}{2} \cdot \frac{\Delta a}{a} = \frac{2 + 5e^2}{1 - e^2} \cdot 3\pi c \quad . \quad . \quad . \quad (14)$$

The effect of the new term is thus to increase the change in the mean motion in the ratio

$$2 + 5e^2 : 2 + 2e^2$$

so that, with the same notation and constants as before, equation (1) of the previous paper becomes

$$\rho a = 4{\cdot}6 \times 10^{-15} \frac{(2 + 5e^2)(1 - e)^{\frac{1}{2}}}{(1 + e)^{\frac{3}{2}}} \cdot \frac{b^2}{q^2} \cdot \frac{1}{\Delta n} \quad . \quad (15)$$

6. For Encke's Comet, $e = 0{\cdot}85$, we now obtain

$$\rho a = 0{\cdot}0093 \text{ gm./cm.}^2 ; \ \mu'/\mu = 0{\cdot}008$$

These figures do not differ very materially from those previously found. They are of the same order, and equally fail to substantiate the suggestion that an acceleration in the mean motion of Encke's Comet can be explained in this particular way. This conclusion, though disappointing, is only what might have been anticipated. Of the forces enumerated in § 2, the components (iii.), (iv.), and (v.), as compared with (ii.), the direct radiation pressure, are of the same order as cometary velocities compared with the velocity of light. Their effect is therefore naturally far smaller, and can only become adequate to explain a conspicuous inequality when the gravitation constant itself is effectively diminished by the back pressure due to radiation. The forces which depend on the velocity of the comet are dissipative, as Professor Poynting notices, and therefore cumulative in effect. But though their influence may thus become of the greatest importance by the lapse of time, it is the conservative component of which evidence must first be expected from direct observation.

7. Since my earlier paper the question of evidence on this point has advanced, and we now know that as long ago as 1891 the late Dr August Svedstrup had been led by his labours on the motion of Comet 1886 I. to the conclusion that the constant of solar attraction was in this case effectively diminished. The result is the more remarkable that the author seems to have been uninfluenced by any hypothesis as to the action of radiation. In place of the Gaussian constant k, he found it necessary to use (A.N., 4062, p. 98)

$$\mathrm{K} = k(1 - 0{\cdot}0000415)$$

which gives

$$\mu'/\mu = 1 - \mathrm{K}^2/k^2 = 0{\cdot}000083$$

Hence

$$a\rho = \frac{3}{4} \cdot \frac{\mathrm{S}}{\mathrm{U}} \cdot \frac{b^2}{\mu} \cdot \frac{10^6}{83}$$

$$= 0{\cdot}89 \text{ gm./cm.}^2$$

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if we adopt $S = 1 \cdot 75 \times 10^6$. This shows that if ρ is about 6, or something like the mean density of the Earth, the diameter of the particles composing the comet is of the order of 3 mm. This result confirms the hypothesis as to the physical constitution of comets on which this investigation rests, and supplies the first numerical estimate of the size of the meteoric bodies which constitute a cometary swarm.

8. Dr Svedstrup's discovery amply justifies the statement before made, that "it is unwarrantable to *assume* that the mean motion and the mean distance are related and not independent elements." This must be borne in mind in all future discussions of the definitive orbits of comets; otherwise a valuable means of gaining an insight into the constitution of these bodies will be neglected. It now appears exceedingly probable that in this direction lies the clue to the explanation of the anomalous motion of Encke's Comet, as suggested in § 3 of my earlier paper.

University Observatory, Oxford:
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Note on a Mechanical Solution of Kepler's Equation.

By H. C. Plummer, M.A.

This note refers to the method of obtaining an approximate solution of Kepler's equation which was proposed by Dr Rambaut (*Monthly Notices*, vol. l. p. 301), and was subsequently described independently by myself (*Monthly Notices*, vol. lvi. p. 317), and again later by Herr B. Gonggrijp (*Ast. Nach.*, 3720). The principle of the method, which is based on the use of a trochoid, is to be found in Newton's *Principia*, book i. prop. 31.

Quite recently Dr Rambaut has proposed a new method which combines great mathematical elegance with considerable practical advantages (*Monthly Notices*, vol. lxvi. p. 519). His paper suggests that it may not be out of place if I describe some modifications in the application of the trochoid principle which occurred to me, by a curious coincidence, about the same time.

In the accompanying figure, A B C is a flat circular disc of suitable radius, mounted on a board so as to be capable of turning about its centre O. Fixed to the board and flush with the surface of the disc are wooden pieces Y Y, N N, which serve as guides to a scale S S, which, with bevelled edges, can slide between them. The point E, marked with a pointer, is the foot of the perpendicular from O on the lower edge of the scale. Attached near B, and passing round the edge of the disc, is a thin flexible metallic tape F, the other end of which is attached to the end of the scale S S. This tape is kept taut by another tape B C P passing round the disc in the opposite direction and fastened to a weak spring or, much better, to a hanging weight.